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A THEORY OF OPTIMAL AGENDA DESIGN

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ABSTRACT

This paper formalizes the problem of designing optimal agendas for voting over finite alternative spaces, when voters are assumed to be "naive," (i.e., they do not vote strategically). The class of agendas considered here is quite broad, and includes, as special cases, such methods as pairwise voting, sequential and elimination procedures, partitioning schemes, and all binary procedures. Given individual preferences over the basic alternative space, and various assumptions about how individuals choose between subsets of alternatives, one can then formalize the problem of designing agendas as a dynamic programming problem and solve for optimal agendas, i.e., agendas having either the highest probability of leading to a given alternative or having the highest expected utility to the agenda setter. Illustrations are given showing how the methods can be applied in specific examples.

A THEORY OF OPTIMAL AGENDA DESIGN*

I. INTRODUCTION

Consider a group of voters who are faced with a choice they must make among some set of feasible alternatives. One individual (or group of individuals) is given the authority to set the agenda for the meeting at which the issue will be decided. By an "agenda," we mean a specification of exactly what questions will be put to a vote, and in what order. At the meeting, the agenda is assumed to be the order of the day, and we assume all motions arising from the floor will be ruled out of order. The agenda setter is assumed to have complete knowledge of the preferences of all voters while the rest of the voters are assumed not to have enough information on the preferences of other voters or the structure of the agenda to be able to make use of it in any strategically advantageous manner.

The above scenario might be a reasonable representation of the state of information and control available in a fairly large organization, which meets relatively infrequently, under time constraints, and in which the chairman (or agenda setter) is able to poll the membership prior to meeting time. Stockholders meetings, trade

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associations, interest groups and clubs would be examples of such organizations. The subject of this paper is how to design an optimal agenda in such circumstances, where by optimal agenda we mean an agenda which maximizes the expected utility to the agenda setter.

It has long been known that in voting situations where a group must select an alternative out of a large (more than two element) alternative space, that the agenda, or order of consideration of alternatives, can itself be a factor in determination of the final outcome. See e.g., Black [1978], Carrol [1873], and Farquharson [1969]. Recent papers by Plott and Levine [1977], and [1978], demonstrate convincingly the existence of such an effect in an experimental setting.

Although the importance of the agenda has been generally recognized, the extent to which the agenda can affect the final outcome in a given situation has not been satisfactorily answered. This is because there only exist, at present, models for determining the outcome of a <u>given</u> agenda (see Plott and Levine [1978]) and there are no systematic methods for determination of the "best" agenda for achieving a given outcome. In principle, once one has a model which determines the outcome of a <u>given</u> agenda, one could determine the best agenda by an exhaustive search, as long as the alternative space is finite. Unfortunately, the number of possible agendas is generally extremely large, even for relatively small problems, making this approach prohibitive.

This paper formalizes the agenda design problem as a dynamic programming problem. Using this approach, it is possible to solve for optimal agendas in problems involving a moderate number of basic

alternatives. In any actual application, the structure of the alternative space or the particular parliamentary rules in force would imply that only certain types of motions are admissible. Here we have only introduced those restrictions which would be met in most cases. But imposition of such additional restrictions on the feasible agendas should make the procedures of this paper computationally feasible on larger problems.

Any model of the effect of the agenda must make assumptions about how individuals vote at various stages of the agenda. In the existing models two approaches have been taken, which vary according to whether they assume voters are "sophisticated" or "naive."

The assumption of sophisticated voters, originally developed by Farquharson [1969], is equivalent to assuming a sort of super rationality on the part of the voters. Voters are assumed to have perfect information about everyone else's preferences and about the structure of the agenda. They then adopt optimal strategies from a game theoretic point of view in the resulting n person game. Such a game can also be expressed as a multi stage game, and if the agenda is binary (i.e., every vote involves only two possible outcomes), it can be solved to determine the (unique) alternative that will be selected. This type of analysis establishes that with sophisticated voters, binary agendas must lead to alternatives in the top cycle set. (See McKelvey and Niemi [1978]).

In most empirical applications voters usually do <u>not</u> have the information necessary to behave as sophisticated voters, further even if they have this information, time pressures and the complexities of making the necessary calculations preclude the possibility of sophisticated behavior. Further, the experiments of Plott and Levine do not provide any empirical support for sophisticated behavior.

The second approach -- the assumption of naive voting -- takes a much more limited view of the information available to the voter, and hence of the level of rationality of which he is capable. Specifically, at any stage of the agenda, it is assumed that the voter does not have enough information about the preferences of others or of the subsequent agenda to allow him to infer anything about the outcome that will eventually prevail. Thus, his beliefs -- if he has any -- about what will finally happen are independent of the preference configurations of others. Plott and Levine [1978] develop a model of individual behavior in agendas for the naive case. They derive probabilities that a vote in an agenda will go a certain way by assuming that voters can be considered as random variables drawn from three possible decision rules. Although their model is consistent with the experimental data they report, the model is somewhat ad hoc, and there is not really enough data to reject alternative models.

The approach taken in this paper is similar to that of Plott and Levine in that it assumes naive voters. However, rather than modeling this as they do, we take a more standard decision theoretic approach, and assume that each voter is a subjective expected utility maximizer. Thus, at each stage of the agenda, we assume that if a voter is faced with a choice between different sets of alternatives, he has some probability distribution representing his subjective estimate that each of the alternatives in a set will eventually prevail if that set is chosen. The voter then chooses between the sets on the basis of

expected utility. Individuals may differ in their estimates of the probability of given events, and the approach allows for the use of any empirically estimated distribution of such beliefs.

The remainder of the paper is organized as follows: Section 2 gives definitions and notation, introducing in turn, voters, agendas, and the agenda setter. Section 3 formalizes the problem as a dynamic programming problem, and shows how this formulation can be used to obtain simple algorithms for the computation of the value of a given agenda and for the determination of the optimal agenda. Section 4 gives examples illustrating the use of these methods on specific problems, and Section 5 gives conclusions along with suggestions for further modifications of the model.

2. DEFINITIONS AND NOTATION

2.1 An Example

Before proceeding with the formal definitions, we give a short example intended to motivate the particular formalization used here. We consider a board of directors of a company faced with a decision as to whether to build any new plants. For simplicity, there are four alternatives. They are

a = build no plants

b = build plants in both St. Louis and Pittsburgh

c = build only one plant in Pittsburgh

d = build only one plant in St. Louis.

There are several ways in which the board might proceed to consider these alternatives. One way is to propose and vote sequentially on the following motions.

(i) Shall we build a plant in Pittsburgh

(ii) Shall we build a plant in St. Louis.

This procedure can be represented by the voting tree illustrated in Figure 1(a).

Now by changing the order or form of the motions that are made, we can change the form of the agenda, and of the corresponding voting tree. Thus Figure 1b represents the voting tree that results from reversing the order of consideration of the two cities. Figure 2 represents the agenda resulting from the following sequence of motions.

(i) Do we want to construct any new plants

(ii) If so, do we want to construct one or two

(iii) If we want only one, shall it be in Pittsburgh or St. Louis.

A more complicated agenda that might arise from strict adherence to parliamentary procedures is in Figure 3. This arises from the following sequence of motions.

(i) I move we build two new plants, one in Pittsburgh and one in St. Louis.

(ii) I move that we amend the preceding motion to specify only one plant in Pittsburgh

(iii) I move we amend Pittsburgh to St. Louis

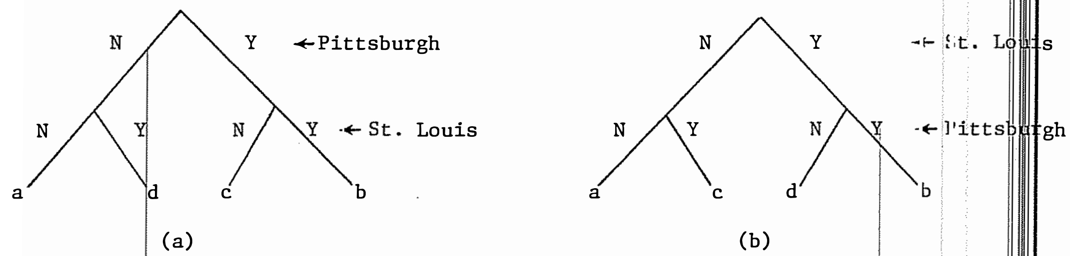


Figure 1

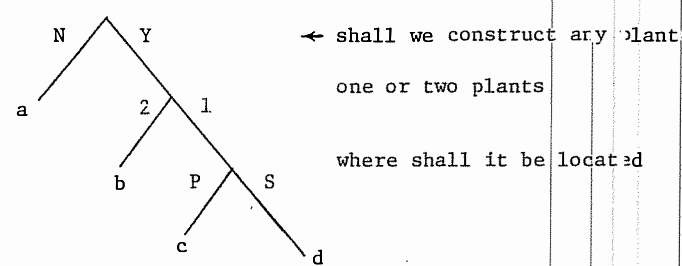


Figure 2

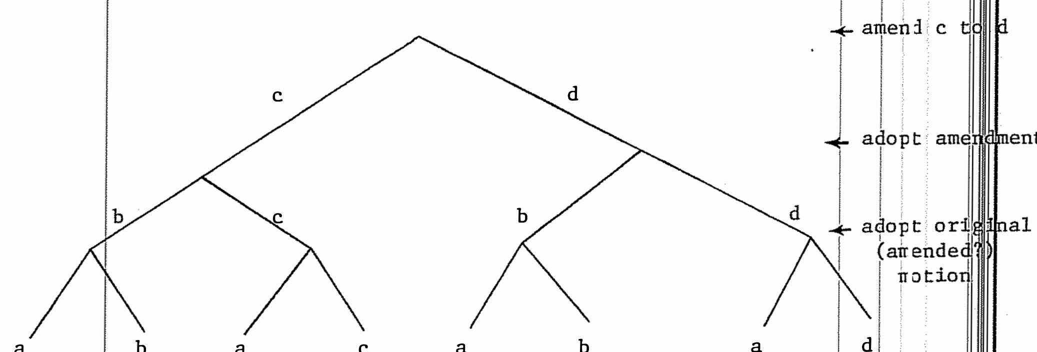


Figure 3

Here, if we follow Robert's Rules of Order, the amendment to the amendment (motion (iii)) is voted on first, then the amendment (motion (ii)), then the original (perhaps amended) motion. This yields the voting tree of Figure 3.

Now, in each of the above agendas, individuals must vote in the early stages of the agenda in the absence of any knowledge of what the outcomes of later motions are. All the voter knows for sure is that if the vote results in the choice of one branch, then the final outcome must be one of the alternatives at the terminal nodes following that branch. So the voters must effectively choose between <u>sets</u> of alternatives. Correspondingly, an agenda can be viewed as a means of successively subdividing a set (of presently feasible alternatives) into smaller subsets of alternatives. Thus, in the agenda of Figure 1(a), before the first motion is voted on, all four alternatives, i.e., {a,b,c,d} are feasible. The first motion proposes to limit the feasible set to either {a,d} or {c,b}, depending on the outcome of the first vote. If the outcome is {a,d}, the next motion, which is also the final motion, proposes to choose either {c} or {d}.

In order to predict what will happen in agendas of the above sort, we first need a model of individual behavior in the early stages of the agenda. This is the subject of Section 2.2. In order to then find optimal agendas, we need to specify, fairly rigorously, the class of admissible motions and agendas. Section 2.3 does this. Finally, to know what is optimal, we have to know the preferences and information available to the agenda setter. Section 2.4 treats this.

2.2 Alternatives, Voters and Preferences

Let $X = \{x_1,\ldots,x_m\}$ be a finite set of <u>alternatives</u>. We let $\underline{X} = \mathcal{P}(X)$ denote the set of all nonempty subsets of X, and $\underline{\underline{X}} = \mathcal{P}(\underline{X})$ be the set of all nonempty subsets of $\underline{X}$. Elements of $\underline{X}$ are then sets of alternatives, and will be denoted S,T, etc. Elements of $\underline{\underline{X}}$ are sets of sets of alternatives, and will be denoted $\underline{S}$, $\underline{T}$, etc. For any finite set A, we let $|A|$ denote the number of elements of A.

We assume that $N = \{1,2,\ldots,n\}$ is a set of <u>voters</u>, such that for each voter $i \in N$, there is a function $u_i : X \to R$ representing voter i's <u>utility function</u> over X. We further assume that for each voter, there is a probability distribution associated with each of the possible sets that could be arrived at, reflecting that voter's estimate of the likelihood of the eventual occurrence of each alternative in that set, conditional on that set being on the floor. More formally, for each $i \in N$ and $S \in \underline{X}$, there is a function $P_{S,i} : S \to R$ satisfying

$$\sum_{x\in S} P_{S,i}(x) = 1 \quad \text{and} \quad P_{S,i}(x) \geq 0 \qquad (2.2.1)$$

for all $x \in S$. For each $i \in N$, then, it follows that we can extend the utility function of voter i to $\underline{X}$ as follows: For any $S \in \underline{X}$,

$$\underline{u}_i(S) = \sum_{x \in S} u_i(x) p_{S,i}(x) \qquad (2.2.2)$$

Now for each $i \in N$, $\underline{u}_i$ defines a binary relation $\underline{R}_i$ on $\underline{X}$ as follows: For any $S,T \in \underline{X}$

$$S\underline{R}_i T \iff \underline{u}_i(S) \geq \underline{u}_i(T) \qquad (2.2.3)$$

In the usual manner, we can let $\underline{P}_i$ and $\underline{I}_i$ represent the strict and indifferent portions of $\underline{R}_i$. Thus, $S\underline{P}_i T \iff (S\underline{R}_i T \text{ and } \sim (T\underline{R}_i S))$, and $S\underline{I}_i T \iff (S\underline{R}_i T \text{ and } T\underline{R}_i S)$. Now, we let $\underline{\Theta}_i$ be the set of all possible binary relations on $\underline{X}$ generated in the above fashion, and let $\underline{\Theta} = \prod_{i=1}^{n} \underline{\Theta}_i$. Elements of $\underline{\Theta}$ are denoted $\underline{R} = (\underline{R}_1, \ldots, \underline{R}_n)$.

It should be emphasized that the above assumptions are equivalent to assuming naive voters. Thus the $p_{S,i}$ are independent of the agenda, (to be defined in the next section) and of the preferences of other voters. This is a reasonable assumption when voters do not have prior information on the structure of the agenda or on the preferences of other voters. If they do have such information, and time to make use of it, this model would not be appropriate. Rather, we would want a model which allowed for sophisticated behavior on the part of voters, as in McKelvey and Niemi [1978].

2.3 The Agenda

We assume there is a social choice function, operating over all subsets of $\underline{X}$, by which decisions at each step of the agenda (to be defined below) are to be made. Specifically, we let $C : \underline{\Theta} \times \underline{\underline{X}} \to \underline{X}$ be a decisive social choice function. I.e., for any $\underline{S} \in \underline{\underline{X}}$, and $R \in \underline{\Theta}$, $C(\underline{R},\underline{S}) \in \underline{S}$. Next, for each $S \in \underline{X}$, we assume there is a set $D_S \subseteq \mathcal{P}(\mathcal{P}(S))$, called the decision set associated with S. An element of D_S is denoted d_S, and is called a motion. The outcome of the motion d_S is the set $C(R, d_S) \in d_S$. The space $\Delta = \prod_{S \in \underline{X}} D_S$ is called the agenda space, and an element of Δ is called an agenda, written δ, with δ_S representing the element of δ pertaining to S.

Interpreting the above definitions, we can think of the set $S \subseteq X$ as the set of alternatives that are still under consideration at a particular point in time. A motion, d_S, is a particular subdivision of S into one or more (perhaps overlapping) subsets of S. Voters then choose among these subsets using the social choice rule, leading to a new set $C(\underline{R},d_S) \in d_S$ which then comes under consideration.

For example, if $S = \{a, b, c\}$, then a motion would be of the form, say, $d_S = \{\{b, c\}, \{a, c\}\}$. This particular motion is equivalent to a motion to "eliminate a in favor of b." If the voters choose the set $\{b, c\}$, then this set is considered next. The decision set, D_S, represents the set of allowable, or "legal" motions given that the set S is on the floor. In a given application, the parliamentary rules would determine what the set D_S looks like. So for example we might have $D_S = \{\{\{b,c\}, \{a\}\},\{\{a,b\},\{c\}\},\{\{a,c\},\{b\}\}\}$ if the rules only allowed for motions which partition the set $\{a,b,c\}$. We consider several possible restrictions on D_S below.

An agenda, δ, is then a specification of a motion, δ_S, (which determines an outcome in δ_S,) for every possible subset $S \subseteq X$. Given an agenda, then, starting at any set $S \subseteq X$, one would then move from set to set, in the manner described above until one eventually arrived at a unique alternative in X (i.e., a singleton set). The concept of an agenda used here is similar to the game theoretic notion of a strategy. The agenda specifies a plan (i.e., a motion) for every contingency that might arise, even though, as we shall see, in any use of the agenda, only a small number of these contingencies will actually arise. Finally, the set Δ simply represents the set of all possible agendas under the rules.

Throughout this paper, we impose a general restriction on Δ. It is assumed that for all $S \in \underline{X}$, D_S satisfies the following condition: for all $S \in \underline{X}$, and $S_i \in \underline{S} \in D_S$,

$$S_i = S \Leftrightarrow |S| = 1 \qquad (2.3.1)$$

This condition requires nontrivial subdivisions to be made whenever S is not a singleton set.

In addition to the above general restriction on Δ, there are a number of restrictions that would be implied by particular parliamentary rules. We will not attempt to model in a detailed fashion such restrictions, but introduce several conditions that are common to many such systems:

(a) (binary procedures) For all $S \in \underline{X}$ with $|S| > 1$, and for all $\underline{S} \in D_S$, $|\underline{S}| = 2$.

All common parliamentary procedures satisfy condition (a), because all business must come to the floor in the form of motions, which are each disposed of one at a time, in order of precedence, by a binary decision on whether to accept or reject the motion. See,

e.g., Robert's Rules of Order [1970]. The examples of section 2.1 illustrate this.

(b) (partitioning procedures) For all $S \in \underline{X}$, and $\underline{S} \in D_S$, $\underline{S}$ partitions S.

In a parliamentary setting, partitioning procedures arise naturally out of motions to divide the question or motions to consider by paragraph or seriatim (see, e.g., Robert's Rules of Order, pp. 230-237). Also, sequential consideration of several separate issues results in partitioning type procedures. The voting trees in Figures 1 and 2 of section 2.1 are of this type.

(c) (sequential or elimination procedures) For all $S \in \underline{X}$ with $|S| > 1$, and for all $S_i \in \underline{S} \in D_S$, $|S - S_i| = 1$.

Although it may not be immediately apparent, in binary procedures this procedure corresponds to the process of voting first between two alternatives, then voting between the winner of this vote and the third alternative, placing the winner of this vote against the fourth alternative, etc. Amendment procedures, such as that illustrated in Figure 3 are of this form,

(d) (successive procedure) (a) and (b) hold, and further for all $S \in \underline{X}$, and $\underline{S} \in D_S$, $|S_i| = 1$ for some $S_i \in \underline{S}$.

This procedure corresponds to procedures frequently used for nominations or "filling blanks," where there is a list of candidates, and one is selected by taking the first to obtain a majority. The voting tree of Figure 2 is of this form.

Although the above restrictions on allowable motions limit considerably the number of agendas in Δ, it should be noted that the agenda space is still generally too large to be able to use simple enumeration for finding "best" agendas. For example, with $|X| = m$, the total number of binary sequential agendas is $\prod_{k=2}^{m} \binom{k}{2}^{\binom{m}{k}}$.
Thus, if $m = 4$, there are 468 such agendas, and for $m = 5$, there are about 4.59×10^9 such agendas. Similarly, the total number of binary partitioning agendas is $\prod_{k=2}^{m} \left(2^{k-1} - 1\right)^{\binom{m}{k}}$. So for $m = 4$, there are 567 such agendas, and for $m = 5$, there are about 1.49×10^{10} such agendas.

There are further restrictions on the allowable motions that are usually implied by parliamentary rules. In particular, germaneness considerations frequently would greatly reduce the number of allowable motions. These type of restrictions are dependent, however, on the structure of the alternative space, and are not dealt with here.

2.4 The Agenda Setter

In addition to the voters, N, we assume there is an additional actor, the agenda setter, who also has preferences over the alternative space. His preferences are represented by the function $u_o : X \to R$ (u_o may coincide with u_i for some $i \in N$).

The problem considered in the next section is that of designing an agenda which is best from the agenda setter's point of view. The answer to this question depends on the amount of information which we assume is available to the agenda setter. Throughout, we always assume that the agenda setter has complete knowledge of the utility functions, u_i, for all voters $i \in N$. However, we assume the agenda setter may or may not have complete knowledge of the individual subjective probability functions $p_{S,i}$. In order to obtain any results, we must assume that the agenda setter himself has some subjective estimate of the likelihood of any given $p_{S,i}$. Now each possible $p_{S,i}$ can be viewed as a point in the corresponding $|S|$ dimensional simplex. We let $\sigma^{|S|}$ represent this simplex. Then, for each $i \in N$, $\delta \in \Delta$, and $S \in \underline{X}$, we assume there is a function $f_{S,i} : \sigma^{|S|} \to R$. If the distribution is continuous, $f_{S,i}$ is the probability density function associated with the agenda setter's subjective distribution of the $p_{S,i}$. If the distribution is discrete, we assume $f_{S,i}$ represents the probability mass function. In the case of full information, $f_{S,i}$ is discrete, and reduces to the following.

$$f_{S,i}(q) = \begin{cases} 1 & \text{if } q_j = p_{S,i}(x_j) \text{ for all } j \\ 0 & \text{otherwise} \end{cases} \tag{2.4.1}$$

In the case of partial information, $f_{S,i}$ would generally be assumed to be a continuous distribution over the simplex, $\sigma^{|S|}$.

Now it should be noted that, although the notation is suppressed, each individual preference relation $\underline{R}_i$ is a function of $p_{S,i}$. Hence $C(\underline{R},\underline{S})$ is also a function of $p_{S,i}$ for all $i \in N$ and $S \in \underline{X}$. Thus, if the $p_{S,i}$ are random variables, then so is $C(\underline{R},\underline{S})$. We use the notation $Pr[C(\underline{R},\underline{S}) = T]$ to denote the probability, under the joint distribution on the $p_{S,i}$, that $C(\underline{R},\underline{S}) = T$.

In this paper, we deal only with two cases: one assumes complete information on the part of the agenda setter, with all voters using an average value rule, the second assumes partial information on the part of the agenda setter, with the agenda setter assuming all subjective probability estimates by the voters are equally likely. We do not claim that either of these cases is a good model of actual behavior, but use them only to illustrate the methods described in this paper. In any actual application, different assumptions on the $f_{S,i}$ can be substituted without affecting any of the results described in the following sections. In fact, in empirical applications it is, of course, very unlikely one would have much information on the $p_{S,i}$. In this case one could use distributions, $f_{S,i}$ estimated exogenously from experimental data. We now describe formally the two cases considered here:

<u>Case I-a</u> (Full Information, Average Utility). For all $i \in N$, and $S \in \underline{X}$, $f_{S,i}$ is as in (2.4.1), with

$$p_{S,i}(x_j) = \begin{cases} \dfrac{1}{|S|} & \text{if } x_j \in S \\ 0 & \text{otherwise} \end{cases} \tag{2.4.2}$$

This corresponds to the case where all voters assume any alternative in S is equally likely, and hence choose between two sets on the basis of which has the highest average utility. For case II, the case of partial information, we consider the following model:

<u>Case II-a</u> (Partial Information, Uniform Prior). For all $i \in N$, $S \in \underline{X}$, and $p,q \in \sigma^{|S|}$,

$$f_{S,i}(p) = f_{S,i}(q) \tag{2.4.3}$$

Case II-a corresponds to the case when the agenda setter assumes that any subjective distribution over the alternatives in $\sigma^{|S|}$ is equally likely. A more reasonable assumption than either of these extreme cases might be to assume that $f_{S,i}$ could be a general Dirichlet distribution on $\sigma^{|S|}$. Both models above then become limiting cases. For the purposes of tractability, we confine ourselves in our illustrations to the above two models.

3. AGENDA DESIGN AS A DYNAMIC PROGRAMMING PROBLEM

3.1 <u>The Value of a Given Agenda</u>

The first question we address is that of determining the "value" of a <u>given</u> agenda. Thus, given an agenda $\delta \in \Delta$, we wish to find a function $v_\delta : \underline{X} \to R$, such that for all $S \in \underline{X}$, $v_\delta(S)$ represents (in a sense to be made precise below) the expected utility to the agenda setter of being at S under the agenda δ. To answer this question, we let V be the space of all real valued functions on $\underline{X}$. Then, for each $S \in \underline{X}$, $d_S \in D_S$, and $v \in V$, define the "return function," h, as follows:

$$h(S,d_S,v) = \begin{cases} \sum_{S_i \varepsilon d_S} \Pr[C(\underline{R},d_S) = S_i]v(S_i) & \text{if } |S| > 1 \\ u_o(x) \text{ where } x \varepsilon S & \text{if } |S| = 1 \end{cases} \tag{3.1.1}$$

In the case of full information on the part of the agenda setter, (3.1.1) reduces to

$$h(S,d_S,v) = \begin{cases} v(C(\underline{R},d_S)) & \text{if } |S| > 1 \\ u_o(x) \text{ where } x \varepsilon S & \text{if } |S| = 1 \end{cases} \tag{3.1.2}$$

The function $h(S,d_S,v)$ represents the value the agenda setter should assign to the set S if this value is to represent the expected value of S, and is also to be consistent with the values $v(S_i)$ assigned to the sets $S_i \varepsilon d_S$. Now in general, given a particular $\delta \varepsilon \Delta$ and $v \varepsilon V$, v(S) need not be equal to $h(S,\delta_S,v)$ for all S ε $\underline{X}$, because the values assigned by v to different sets may not be consistent with each other under the agenda δ. An appropriate value for the agenda δ is a function v_δ which is always consistent in the above sense. Thus, such a function should satisfy

$$v_\delta(S) = h(S,\delta_S,v_\delta) \tag{3.1.3}$$

for each S ε $\underline{X}$.

Now it is easily verified that h satisfies the n stage contraction and monotonicity assumptions for dynamic programming problems (see Appendix). Hence for each agenda $\delta \varepsilon \Delta$, there is a unique element $v_\delta \varepsilon V$ satisfying (3.1.3). (See Denardo, p. 166).

Now as discussed in Denardo, v_δ can be computed iteratively as follows: Define, for each $\delta \varepsilon \Delta$, the function $H_\delta : V \to V$ by

$$[H_\delta(v)](S) = h(S,\delta_S,v) \tag{3.1.4}$$

For any $v_0 \varepsilon V$, define $v_i = H_\delta(v_{i-1})$ for all i > 0. Then $v_i \to v_\delta$, where the convergence is exact after some finite number of steps. In this application, if we set $v_0(S) = 0$ for all S ε $\underline{X}$, the above algorithm corresponds to starting with the last stage of the agenda and working backwards, computing the value of each subset in terms of the (already computed) values of the succeeding sets.

3.2 The Optimal Agenda

Of more interest is the optimal agenda. Formally, we want to find (if it exists) an agenda, $\delta^* \varepsilon \Delta$, and the corresponding function $v^* \varepsilon V$ such that for all S ε $\underline{X}$,

$$v^*(S) = v_{\delta^*}(S) = \sup_{\delta \varepsilon \Delta} v_\delta(S) \tag{3.2.1}$$

Again, since the n stage contraction assumption is satisfied, it follows that v^* exists, is unique, and can be attained with an agenda δ^* which can be found by a simple iterative procedure. Specifically, following Denardo, define the function $A : V \to V$ by

$$(Av)(S) = \sup_{d_S \varepsilon D_S} h(S,d_S,v) \qquad (3.2.2)$$

The mapping A has a unique fixed point, v^*, which corresponds to the solution to (3.2.1). This can be obtained iteratively by letting $v_o \varepsilon V$ be any initial function, and then setting $v_i = Av_{i-1}$ for $i \geq 1$. It follows that $v_i \to v^*$. Again, in this application, convergence is exact after a finite number of steps, and we take δ^* to be any agenda which attains v^*. I.e., such that $H_{\delta^*}(v^*) = v^*$.

4. EXAMPLES

We consider two examples. The first example has three voters and five alternatives. The second has nineteen voters, and eight alternatives, and corresponds to an example run experimentally by Plott and Levine [1974]. Preferences of the voters in these two examples are given in Tables 1 and 2, respectively. As in Plott and Levine, we consider here only binary agendas, and assume that the choice function, at each stage, corresponds to majority rule. Specifically, if $\underline{S} = \{S_k,S_j\}$,

$$C(\underline{R},\underline{S}) = S_k \iff |\{i \varepsilon N \mid S_k \underline{P}_i S_j\}| > |\{i \varepsilon N \mid S_j \underline{P}_i S_k\}| \qquad (4.1)$$

The first example has a completely cyclic social ordering, as illustrated in Table 1, while the second example is transitive, with the exception of an intransitive indifference between c, e and h. Thus, we expect that a sequential agenda can attain any outcome in Example 1, but only alternative f in Example 2. A partitioning or other nonsequential procedure would be needed in Example 2 to get alternatives other than f.

In the computation of the optimal agendas for these examples, the agenda setters preferences are assumed to be as given in the last rows of Tables 1 and 2. In both examples, the agenda setters preferences are picked to represent the inverse of the "natural" social order, so that we can investigate how far down in the social order it is possible to get with the optimal agenda. (In example 1, since the social order is cyclic, the "natural" social order is assumed to be that given by the sum of the utilities.) We first consider the full information model for both these examples and then the partial information model.

Case I: Full Information, Average Utility

Tables 3 and 4 give the optimal sequential agenda and partitioning agenda, respectively, for example 1. The tables describe the complete agenda, indicating the value, $v(S_i)$ of each alternative, as well as the optimal pair of sets $\underline{S}_i = \{S_j,S_k\} \varepsilon D_{S_i}$ into which S_i should be divided. The columns $Pr(S_j)$ and $Pr(S_k)$ indicate the outcome of this division. $Pr(S_j) = 1$ indicates that the division will result in S_j, while $Pr(S_j) = 0$ indicates that the division will result in S_k (so $Pr(S_k) = 1$). The figures at the bottom of each table display the "reachable" portion of the agenda, i.e., those sets which can conceivably be reached by some subdivision of the entire set S = X.

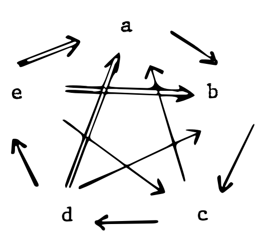
TABLE 1

Utility of Voters over X

Voter #	a	b	c	d	e
1	12	2	5	9	1
2	9	13	0	1	2
3	7	10	15	0	1
0	2	5	10	20	26

Majority Relation over X

(Double arrow indicates Pareto preference.)



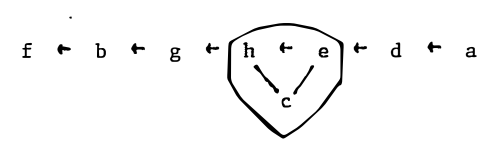
Example 1. Preferences of Voters and Social Ordering

TABLE 2

Utility of Voters over X+

Voter #	FFF a	FFC b	FFA c	FFFF d	FFFC e	FFFA f	FFCC g	FFAA h
1	0.50	6.00	1.00	0.25	0.75	8.00	5.50	5.00
2	0.25	0.25	0.25	0.25	5.00	4.50	8.00	0.25
3	0.55	8.00	0.49	0.40	7.80	0.45	7.90	0.29
4	3.80	7.50	3.50	0.90	4.00	3.80	8.00	1.00
5	3.00	2.00	5.00	4.00	7.00	8.00	6.00	8.00
6	6.00	8.00	4.00	3.00	5.00	3.00	7.00	3.00
7	4.00	6.00	6.00	5.00	6.00	7.00	8.00	7.00
8	1.00	1.00	8.00	1.00	1.00	8.00	1.00	8.00
9	2.00	1.00	3.00	5.00	4.00	8.00	6.00	7.00
10	2.00	3.00	4.00	2.00	5.00	6.00	7.00	8.00
11	1.00	7.50	7.40	2.00	2.00	8.00	7.00	6.00
12	2.00	8.00	2.00	7.00	6.00	5.00	1.00	7.00
13	7.00	6.00	5.00	7.00	1.00	8.00	2.00	3.00
14	3.00	2.00	2.00	7.00	1.00	8.00	7.00	6.00
15	1.00	7.00	7.00	2.00	2.00	8.00	4.40	3.60
16	7.00	7.00	1.00	2.00	3.00	3.00	2.00	8.00
17	1.00	2.00	8.00	7.00	3.00	7.00	6.00	5.00
18	7.00	7.00	6.00	8.00	5.00	5.00	1.00	6.00
19	7.00	7.00	6.00	1.00	1.00	8.00	6.00	5.00
0	8.00	2.00	5.00	7.00	6.00	1.00	3.00	4.00

Majority Relation over X



Example 2. Preferences of Voters and Social Ordering

+This example is from Plott and Levine (1974).

Note that the optimal sequential agenda corresponds, as we would expect, to the unique cycle ending at e which passes through all points in X. Thus the first division corresponds to a vote between a and b. When b wins, the second division is between b and c, etc. As expected, a partitioning agenda in this case can not do as well as the sequential agenda. The partitioning agenda only can obtain d, while the sequential agenda can attain e.

Table 5 gives the reachable portion of the optimal partitioning agenda for example 2. The agenda obtains c with probability $\frac{1}{2}$ and h with probability $\frac{1}{2}$. These are the fourth and fifth ranked alternatives for the agenda setter, and represent the lowest alternatives in the social order which can be obtained by any agenda under model I-a (see Table 8).

Case II: Partial Information, Uniform Prior

For the case of partial information, we assume the uniform prior model, Case II-a, as defined in Section. 2. In order to solve for this model, we need to determine $\Pr[C(\underline{R},d_S) = S_j]$ for all $S \in \underline{X}$, and $S_j \in d_S \in D_S$. Since the closed form expression for this probability would be quite complex, and would be dependent on the particular form of the distribution $f_{S,i}$ of the $p_{S,i}$ which is assumed, we elected to compute these values using Monte Carlo methods. This was done as follows: For each $S \in \underline{X}$, and $d_S \in D_S$, a <u>trial</u> consisted of a drawing of $p_{S_j,i}$ for each $i \in N$ and $S_j \in d_S$. For each trial, we can then compute $\underline{u}_i(S_j)$ for each $i \in N$ and $S_j \in d_S$. From this information, we can compute $C(\underline{R},d_S)$ for this particular trial. Let $C^k(\underline{R},d_S)$

TABLE 3

OPTIMAL SEQUENTIAL AGENDA FOR EXAMPLE 1 UNDER MODEL I-1
(FULL INFORMATION AVERAGE UTILITY MODEL)

Set # i	Value $v(S_i)$	Next Division S_j	S_k	Probability $\Pr(S_j)$	$\Pr(S_k)$	Alternatives in Set a	b	c	d	e
1	26.000					0	0	0	0	1
2	20.000					0	0	0	1	0
3	26.000	2	1	0.00	1.00	0	0	0	1	1
4	10.000					0	0	1	0	0
5	10.000	4	1	1.00	0.00	0	0	1	0	1
6	20.000	4	2	0.00	1.00	0	0	1	1	0
7	26.000	5	3	0.00	1.00	0	0	1	1	1
8	5.000					0	1	0	0	0
9	5.000	8	1	1.00	0.00	0	1	0	0	1
10	5.000	8	2	1.00	0.00	0	1	0	1	0
11	5.000	10	3	1.00	0.00	0	1	0	1	1
12	10.000	8	4	0.00	1.00	0	1	1	0	0
13	10.000	12	5	1.00	0.00	0	1	1	0	1
14	20.000	10	6	0.00	1.00	0	1	1	1	0
15	26.000	11	7	0.00	1.00	0	1	1	1	1
16	2.000					1	0	0	0	0
17	2.000	16	1	1.00	0.00	1	0	0	0	1
18	2.000	16	2	1.00	0.00	1	0	0	1	0
19	2.000	18	3	1.00	0.00	1	0	0	1	1
20	2.000	16	4	1.00	0.00	1	0	1	0	0
21	2.000	20	5	1.00	0.00	1	0	1	0	1
22	2.000	20	6	1.00	0.00	1	0	1	1	0
23	2.000	22	7	1.00	0.00	1	0	1	1	1
24	5.000	16	8	0.00	1.00	1	1	0	0	0
25	5.000	24	9	1.00	0.00	1	1	0	0	1
26	5.000	24	10	1.00	0.00	1	1	0	1	0
27	5.000	26	19	1.00	0.00	1	1	0	1	1
28	10.000	20	12	0.00	1.00	1	1	1	0	0
29	10.000	28	21	1.00	0.00	1	1	1	0	1
30	20.000	22	14	0.00	1.00	1	1	1	1	0
31	26.000	23	15	0.00	1.00	1	1	1	1	1

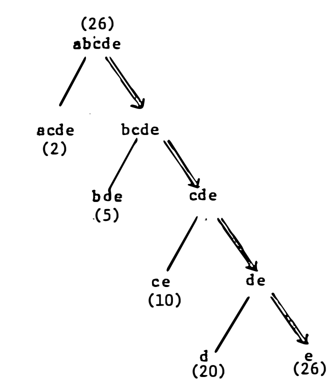


TABLE 4

OPTIMAL PARTITIONING AGENDA FOR EXAMPLE 1
(FULL INFORMATION AVERAGE UTILITY MODEL)

Set # i	Value $v(S_i)$	Next Division S_j	S_k	Probability $Pr(S_j)$	$Pr(S_k)$	Alternatives in Set a	b	c	d	e
1	26.000					0	0	0	0	1
2	20.000					0	0	0	1	0
3	26.000	2	1	0.00	1.00	0	0	0	1	1
4	10.000					0	0	1	0	0
5	10.000	4	1	1.00	0.00	0	0	1	0	1
6	20.000	4	2	0.00	1.00	0	0	1	1	0
7	20.000	6	1	1.00	0.00	0	0	1	1	1
8	5.000					0	1	0	0	0
9	5.000	8	1	1.00	0.00	0	1	0	0	1
10	5.000	8	2	1.00	0.00	0	1	0	1	0
11	5.000	10	1	1.00	0.00	0	1	0	1	1
12	10.000	8	4	0.00	1.00	0	1	1	0	0
13	10.000	12	1	1.00	0.00	0	1	1	0	1
14	10.000	12	2	1.00	0.00	0	1	1	1	0
15	20.000	9	6	0.00	1.00	0	1	1	1	1
16	2.000					1	0	0	0	0
17	2.000	16	1	1.00	0.00	1	0	0	0	1
18	2.000	16	2	1.00	0.00	1	0	0	1	0
19	2.000	18	1	1.00	0.00	1	0	0	1	1
20	2.000	16	4	1.00	0.00	1	0	1	0	0
21	2.000	20	1	1.00	0.00	1	0	1	0	1
22	2.000	20	2	1.00	0.00	1	0	1	1	0
23	20.000	17	6	0.00	1.00	1	0	1	1	1
24	5.000	16	8	0.00	1.00	1	1	0	0	0
25	5.000	24	1	1.00	0.00	1	1	0	0	1
26	5.000	24	2	1.00	0.00	1	1	0	1	0
27	5.000	26	1	1.00	0.00	1	1	0	1	1
28	5.000	24	4	1.00	0.00	1	1	1	0	0
29	10.000	17	12	0.00	1.00	1	1	1	0	1
30	10.000	18	12	0.00	1.00	1	1	1	1	0
31	20.000	25	6	0.00	1.00	1	1	1	1	1

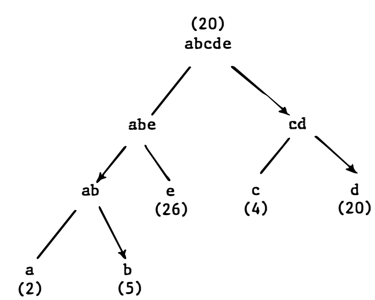


TABLE 5

REACHABLE PORTION OF OPTIMAL PARTITIONING AGENDA
FOR EXAMPLE 2 UNDER MODEL I-a
(FULL INFORMATION AVERAGE UTILITY MODEL)

Set # i	Value $v(S_i)$	Next Division S_j	S_k	Probability $Pr(S_j)$	$Pr(S_k)$	Alternatives in Set a	b	c	d	e	f	g	h
255	4.500	247	8	1.00	0.00	1	1	1	1	1	1	1	1
247	4.500	231	16	1.00	0.00	1	1	1	1	0	1	1	1
231	4.500	198	33	0.00	1.00	1	1	1	0	0	1	1	1
198	3.000	196	2	0.00	1.00	1	1	0	0	0	1	1	0
196	2.000	132	64	0.00	1.00	1	1	0	0	0	1	0	0
132	1.000	128	4	0.00	1.00	1	0	0	0	0	1	0	0
128	8.000	126	1	0.00	1.00	1	0	0	0	0	0	0	0
64	2.000	64	0	1.00	0.00	0	1	0	0	0	0	0	0
33	4.500	32	1	.50	.50	0	0	1	0	0	0	0	1
32	5.000	32	0	1.00	0.00	0	0	1	0	0	0	0	0
16	7.000	16	0	1.00	0.00	0	0	0	1	0	0	0	0
8	6.000	8	0	1.00	0.00	0	0	0	0	1	0	0	0
4	1.000	4	0	1.00	0.00	0	0	0	0	0	1	0	0
2	3.000	2	0	1.00	0.00	0	0	0	0	0	0	1	0
1	4.000	1	0	1.00	0.00	0	0	0	0	0	0	0	1

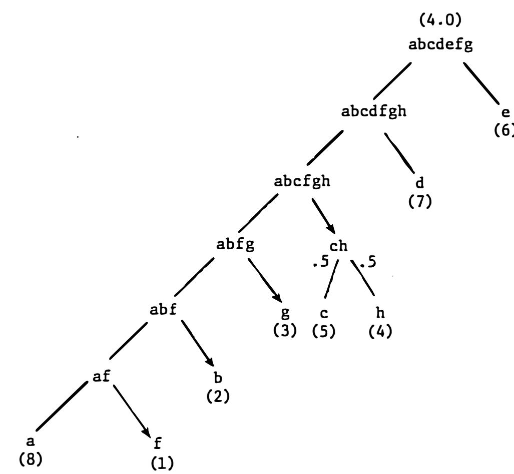


TABLE 6

OPTIMAL PARTITIONING AGENDA FOR EXAMPLE 1
UNDER MODEL II-a
(PARTIAL INFORMATION, UNIFORM PRIOR)

Set # i	Value $v(S_i)$	Next Division S_j	Next Division S_k	Probability $Pr(S_j)$	Probability $Pr(S_k)$	Alternatives in Set a	b	c	d	e
1	26.000					0	0	0	0	1
2	20.000					0	0	0	1	0
3	26.000	2	1	0.00	1.00	0	0	0	1	1
4	10.000					0	0	1	0	0
5	10.000	4	1	1.00	0.00	0	0	1	0	1
6	20.000	4	2	0.00	1.00	0	0	1	1	0
7	20.510	6	1	0.91	0.08	0	0	1	1	1
8	5.000					0	1	0	0	0
9	5.000	8	1	1.00	0.00	0	1	0	0	1
10	5.000	8	2	1.00	0.00	0	1	0	1	0
11	5.525	10	1	0.97	0.02	0	1	0	1	1
12	10.000	8	4	0.00	1.00	0	1	1	0	0
13	10.000	12	1	1.00	0.00	0	1	1	0	1
14	10.750	12	2	0.92	0.07	0	1	1	1	0
15	14.750	9	6	0.35	0.65	0	1	1	1	1
16	2.000					1	0	0	0	0
17	2.000	16	1	1.00	0.00	1	0	0	0	1
18	2.000	16	2	1.00	0.00	1	0	0	1	0
19	2.360	18	1	0.98	0.01	1	0	0	1	1
20	2.000	16	4	1.00	0.00	1	0	1	0	0
21	4.920	17	4	0.63	0.36	1	0	1	0	1
22	2.720	20	2	0.96	0.04	1	0	1	1	0
23	9.830	17	6	0.56	0.43	1	1	1	1	1
24	5.000	16	8	0.00	1.00	1	1	0	0	0
25	5.000	24	1	1.00	0.00	1	1	0	0	1
26	5.000	24	2	1.00	0.00	1	1	0	1	0
27	5.000	26	1	1.00	0.00	1	1	0	1	1
28	6.400	24	4	0.72	0.28	1	1	1	0	0
29	7.900	25	4	0.42	0.58	1	1	1	0	1
30	7.625	24	6	0.82	0.17	1	1	1	1	0
31	12.575	25	6	0.49	0.51	1	1	1	1	1

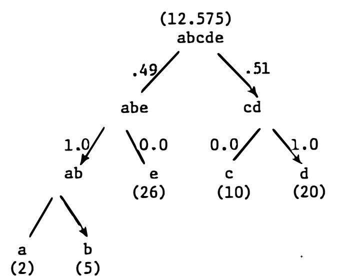
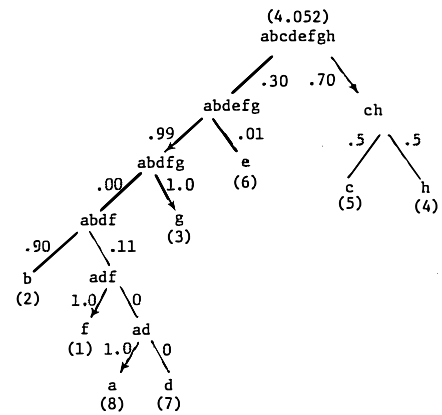


TABLE 7

REACHABLE PORTION OF OPTIMAL PARTITIONING AGENDA
FOR EXAMPLE 2 UNDER MODEL II-a
(PARTIAL INFORMATION - UNIFORM PRIOR MODEL)

Set # i	Value $v(S_i)$	Next Division S_j	Next Division S_k	Probability $Pr(S_j)$	Probability $Pr(S_k)$	Alternatives in Set a b c d e f	g h
255	4.052	222	33	.30	.70	1 1 1 1 1 1	1 1
222	3.030	214	8	.99	.01	1 1 0 1 1 1	1 0
214	3.000	212	2	0.00	1.00	1 1 0 1 0 1	1 0
212	1.895	148	64	.11	.90	1 1 0 1 0 1	0 0
148	1.000	144	4	0.00	1.00	1 0 0 1 0 1	0 0
144	7.000	128	16	0.00	1.00	1 0 0 1 0 0	0 0
128	8.000	128	0	1.00	0.00	1 0 0 0 0 0	0 0
64	2.000	64	0	1.00	0.00	0 1 0 0 0 0	0 0
33	4.500	32	1	.50	.50	0 0 1 0 0 0	0 1
32	5.000	32	0	1.00	0.00	0 0 1 0 0 0	0 0
16	7.000	16	0	1.00	0.00	0 0 0 1 0 0	0 0
8	6.000	8	0	1.00	0.00	0 0 0 0 1 0	0 0
4	1.000	4	0	1.00	0.00	0 0 0 0 0 1	0 0
2	3.000	2	0	1.00	0.00	0 0 0 0 0 0	1 0
1	4.000	1	0	1.00	0.00	0 0 0 0 0 0	0 1



represent the value of $C(\underline{R},d_S)$ on the k^{th} trial. If we draw K trials, as above, an estimate of $Pr[C(\underline{R},d_S) = S_j]$ can then be obtained by taking

$$\hat{Pr}[C(\underline{R},d_S) = S_j] = \frac{|\{k \mid C^k(\underline{R},d_S) = C_j\}|}{K} \tag{4.2}$$

Using these estimated values as the true values, this allows computation of the return function as in (3.1.1). The rest of the analysis proceeds as before. The accuracy of the optimal agenda using this approach will clearly depend on the accuracy of the estimates of the $\hat{Pr}[C(\underline{R},d_S) = S_j]$. We can get these estimates as close as desired to the true values by taking a large enough K. In the results reported here, we take K = 200.

Tables 6 and 7 give the optimal partitioning agenda, for examples 1 and 2 respectively under Case II. Note that the reachable portion of the agenda for example 1 stays the same as it was under Case I, although the value of the agenda has decreased from 20 to 12.575, reflecting the probability that the first step of the agenda will fail. In Example 2, note that the optimal agenda still goes after c and h, but this is now the first order of business, where it has a relatively good probability of success. The likelihood of this step succeeding under the uniform prior model is only .7. In the event this fails, the agenda then takes a long shot at e (with utility of 6), and in the event this fails, goes after g to get it with certainty.

Finally, Table 8 gives the probability under an optimal

TABLE 8

Probability of Attaining each Alternative with Optimal Agenda for that Alternative

Alternative	F ←	B ←	G ←	H	C	E ←	D ←	A
Value - Case I-a	1.00	1.00	1.00	1.00	.50	0.00	0.00	0.00
Value - Case II-a	1.00	1.00	1.0	.98	.34	.025	.0086	0.00

agenda of being able to obtain each of the alternatives in example 2. (Thus, the entries in this table represent the value of an optimal agenda when the agenda setter assigns 1 to the given alternative, and 0 to all other alternatives).

5. CONCLUSION

The dynamic programming approach of this paper allows for constructing optimal agendas without the necessity of performing a complete search of the agenda space. Nevertheless, without further refinement or simplification, even with this approach, the technique is limited to be able to deal with small to moderate sized problems. The computer program which was used to calculate the optimal agendas in the previous section handles quite easily problems of the size of Example 1.

Also, in Case I-a, problems of the size of example 2, which involve nineteen voters and eight alternatives are handled easily (about 95 seconds of CPU time on an IBM 370). However, under Case II-a, this example requires about 25 minutes of CPU time. The large amount of time involved in solving Case II is due to the fact that Monte Carlo procedures are used to compute the $Pr[C(\underline{S},\underline{R}) = S_j]$. Thus, for <u>each</u> $S \in \underline{X}$, and $d_S \in D_S$, two hundred draws are made from a random number generator for <u>each</u> $i \in N$ and $S_j \in d_S$ to determine $Pr[C(d_S,\underline{R}) = S_j]$. Obviously, closed form expressions for estimation of these probabilities would considerably reduce the computation time when large numbers of voters are involved. It is possible that the central limit theorem could be applied in such instances to obtain

estimates of thes probabilities as long as $f_{S,i}$ is independent of i.

Even if closed form expressions for the $Pr[C(S\ \underline{R}) = S_j]$ can be obtained, the computation time is still great enough to imply that twelve to fifteen alternatives would be a limit on the size of the problem that could be handled. Two points should be made in relation to this. First, the current version of the computer program makes no attempt at efficiency. The methods of Section 3 are applied directly, with no attempt to improve on them. Improvements could be made, for example by using branch and bound procedures to do the search implied by equation (3.2.2). This should allow for quickly discarding motions in D_S which obviously can do no better than an already discovered element, d_S. Second, in any actual application, it is likely that there would be additional restrictions on the set of allowable motions, D_S, than those used here. In particular, most partitioning schemes would not allow partitioning according to nonsensical or overly complex criterion. If there are a number of distinct issues, partitioning schemes would generally be required to partition only according to these issues. Similar considerations would apply to the other procedures. Such limitations on the D_S have not been considered here, because they they are dependent on the structure of the issue space. However there is no reason such limitations could not be incorporated. These limitations would drastically reduce the size of the D_S, and since compulation time seems to be linearly related to the number of elements in the D_S, this would enable the procedures to be applied to larger alternative sets.

APPENDIX

I. Proof that $[H(v)](S) = h(S,\delta_S,v)$ satisfies the n stage contraction assumption:

It must be shown that for some $N > 0$, that for any $v,w \in V$, $\exists\ c > 0$ such that

$$\rho[H_\delta^N v, H_\delta^N w] \leq c\rho(v,w) \tag{B.1}$$

where $\rho(v,w) = \sup_{S \in \underline{X}} |\ v(S) - w(S)\ |$. In fact, we show a stronger result, namely that $\rho[H_\delta^N v,\ H_\delta^N w] = 0$, i.e.,

$$H_\delta^N v = H_\delta^N w \tag{B.2}$$

for any $v,w \in V$. In particular we will show that (B.2) is satisfied for $N = m$. To prove (B.2), we show that for any $1 \leq k$, if $S \subseteq X$ satisfies $|S| \leq k$, then for any $v,w \in V$

$$[H_\delta^k v](S) \ = \ [H_\delta^k w](S). \tag{B.3.}$$

This is clearly true for $k = 1$, since when $|S| = 1$, say $S = \{x\}$, then $[H_\delta v](S) = u_o(x) = [H_\delta w](S)$. Now assume that (B.3) is true for k, and we show it is true for $k + 1$. To see this, note first

that if $|S| \leq k + 1$, then by (2.2.1), for any $S_i \in \delta_S$, $|S_i| \leq k$. Thus, if $2 \leq |S| \leq k + 1$,

$$\begin{aligned}
[H_\delta^{k+1} v](S) &= [H_\delta(H_\delta^k v)](S) \\
&= h(S,\delta_S,H_\delta^k v) \\
&= \sum_{S_i \in \delta_S} Pr[C(\underline{R},d_S) = S_i] H_\delta^k v(S_i) \\
&= \sum_{S_i \in \delta_S} Pr[C(\underline{R},d_S) = S_i] H_\delta^k w(S_i) \\
&= h(S,\delta_S,H_\delta^k w) = [H_\delta^{k+1} w](S).
\end{aligned} \tag{B.4}$$

If $|S| = 1$, say $S = \{x\}$, then $[H_\delta^{k+1} v](S) = u_o(x) = [H_\delta^{k+1} w](S)$, so

$$[H_\delta^{k+1} v](S) = [H_\delta^{k+1} w](S) \tag{B.5}$$

holds for all $|S| \leq k + 1$. But now, by induction, it follows that (B.3) is true for all k, in particular for $k = m$. I.e., for <u>all</u> $S \subseteq X$, if $|S| \leq m$

$$[H_\delta^m v](S) = [H_\delta^m w](S). \tag{B.6}$$

But since $|X| = m$, $|S| \leq m$ for all $S \subseteq X$. Hence (B.5) is true for all $S \subseteq X$, so

$$H_\delta^m v = H_\delta^m w \tag{B.7}$$

Q.E.D.

II. Proof that $[H_\delta(v)](S)$ satifies the monotonicity assumption.

We must show that if $v \geq w$, then $H_\delta v \geq H_\delta w$. So let $v, w, \varepsilon V$ satisfy $v \geq w$. Then for any $S_i \varepsilon \underline{X}$, $v(S_i) \geq w(S_i)$, so for any $S \varepsilon \underline{X}$, if $|S| \geq 2$,

$$[H_\delta v](S) = h(S, \delta_S, v)$$

$$= \sum_{S_i \varepsilon \delta_S} Pr[C(\underline{R}, \delta_S) = S_i] v(S_i)$$

$$\geq \sum_{S_i \varepsilon \delta_S} Pr[C(\underline{R}, \delta_S) = S_i] w(S_i)$$

$$= [H_\delta w](S) \tag{B.8}$$

Further, if $|S| = 1$, then $[H_\delta v](S) = [H_\delta w](S)$, so

$$[H_\delta v](S) \geq [H_\delta w](S) \tag{B.9}$$

for all $S \varepsilon \underline{X}$.

Thus $H_\delta v \geq H_\delta w$ as we wanted to show.

Q.E.D.

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